I. SUMMARY OF 3π MC

A. Formalism

The $3\pi = \pi^+(1)\pi^-(3)\pi^+(2)$ events were computed from the following distribution

$$\frac{dN}{dM_{123}d^2\Omega_{13,2}dM_{13}d^2\Omega_{1,3}d\cos(\theta)} = \text{Phase space}|A_1 + A_2|^2 \quad (1)$$

The Phase space is given by a product of three breakup momenta,

$$\text{Phase space} = k_{123,N}k_{13,2}k_{1,3} \quad (2)$$

$k_{123,N}$ –momentum of the recoil nucleon in the overall c.m. frame,

$$k_{123,N} = \lambda(\sqrt{s}, M_{123}, m_N) = \sqrt{\frac{(s-(M_{123}-m_N)^2)(s-(M_{123}+m_N)^2)}{4s}} \quad (3)$$

$k_{13,2}$ –momentum of the spectator pion ($\pi^+(2)$) in the rest frame of the 3π system

$$k_{13,2} = \lambda(M_{123}, M_{13}, m_\pi) \quad (4)$$

and $k_{1,3}$ –momentum of the $\pi^+(1)$ in the rest frame of the $\pi^+(1)\pi^-(3)$ system.

The two amplitudes A_1 and A_2 correspond to permutations of the two π^+

$$\begin{aligned} A_1 &= A_1(\gamma p \to \rho^0\pi^+(2)n \to \pi^+(1)\pi^-(3)\pi^+(2)n) \\ A_2 &= A_2(\gamma p \to \rho^0\pi^+(1)n \to \pi^+(2)\pi^-(3)\pi^+(1)n) \end{aligned} \quad (5)$$

and describe the $\rho^0\pi^+$ production from a decay of the a_2^+ photoproduced via t-channel π exchange. Photon was set to be 100% linearly polarized in the GJ frame.

10^6 MC events were generated using the Metropolis algorithm randomly selecting M_{123} in the $1.2-2.0$ GeV, M_{13} in the $0.4-0.9$ GeV and the two sets of solid angles $\Omega_{13,2}$ and $\Omega_{1,3}$ in a single $\cos(\theta)$-bin ($\cos(\theta)$ describes Nucleon recoil angle in the overall c.m. frame).

These events are generated using photon spin density matrix in the GJ frame.

B. Plots

1. 3π mass distribution

FIGURES

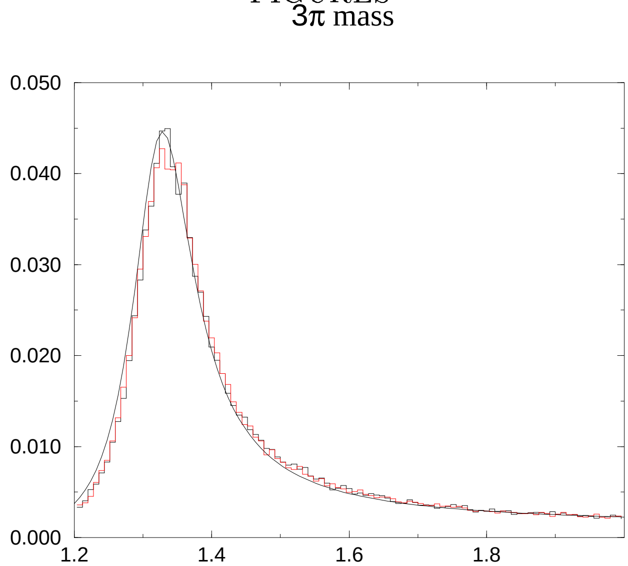

FIG. 1.

Two histograms are shown and correspond to photons polarized either along x or y direction. No difference is found. The solid line is a BW "eyeball" fit with angular momentum barrier factor. The tiny distortion comes from residual dependence on M_{123} (*e.g.* phase space).

2. 2π mass distribution.

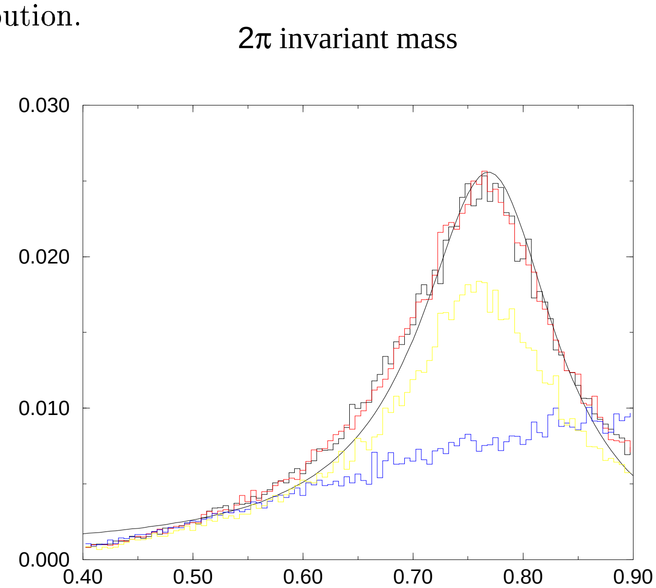

FIG. 2.

Three histograms are displayed. The two close to each other describe the distribution of $\pi^+(1)\pi^-(3)$ mass and, as above, come from two simulations using different photon polarizations. The solid line is the BW "eyeball" fit. The lower histogram with the ρ-like shape is the distribution of the other pair of pions, $\pi^+(2)\pi^-(3)$. The histograms are different because of the phase space factor which depends on M_{13} (and not symmetrically on M_{13} and M_{23}) . The lower histogram shows the distribution of the other pair, $\pi^+(2)\pi^-(3)$ if the amplitude A_2 is turned off, *i.e.* with ρ showing up only in the $\pi^+(1)\pi^-(3)$ combination.

3. $\cos(\theta_{GJ})$ distribution.

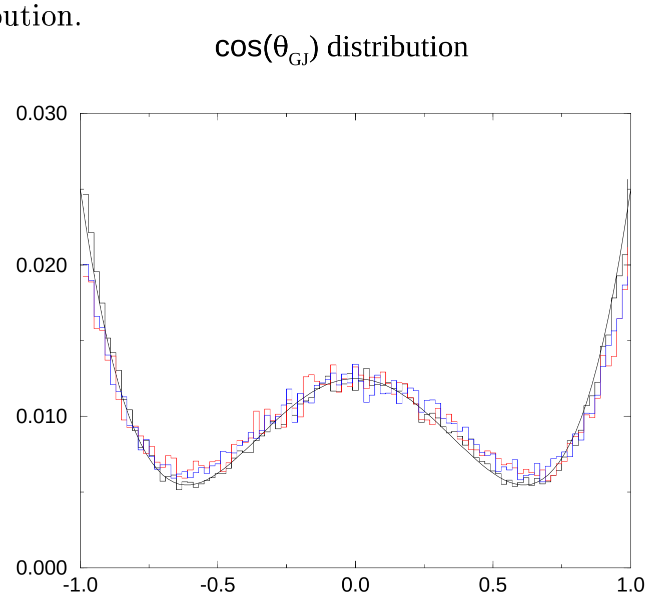

FIG. 3.

Three histograms are shown. One of them fits the solid line. This corresponds to A_2 amplitude turned off. The distribution is then proportional to

$$4\cos(\theta_{GJ})^4 - 3\cos(\theta_{GJ})^2 + 1 \tag{6}$$

(solid line). With both amplitudes turned on, the two histograms (two polarizations) are slightly distorted as compared to the single-diagram prediction.

4.1 ϕ distribution.

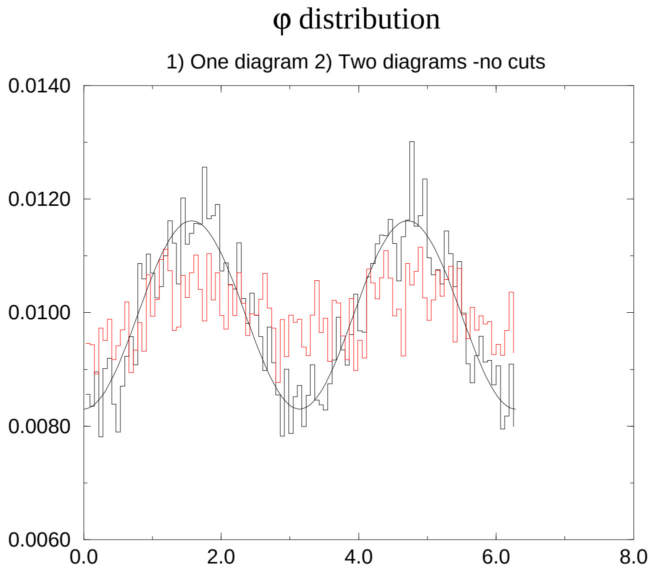

FIG. 4.

This is the azimuthal angle of the $\pi^+(1)$ in the GJ frame. In the first figure photons are polarized along x and two histograms are shown. The solid one has A_2 turned off and it fits the expected (solid line) distribution proportional to

$$7 - 2\cos(\phi)^2 \tag{7}$$

(for y polarization it would be $7 - 2\sin(\phi)^2$). The other histogram (dotted) is the result of the full run (with both amplitudes turned on).

4.2. ϕ distribution continues...

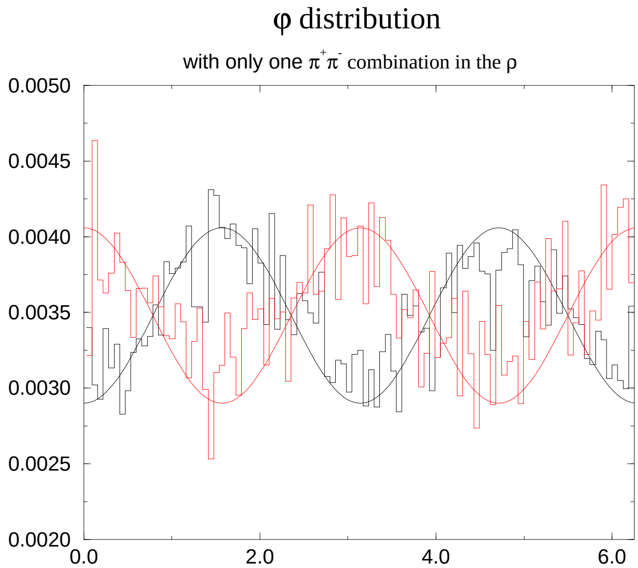

FIG. 5.

In the full run a cut is imposed on the "other", $\pi^+(2)\pi^-(3)$ pion pair. In the histogram only these events are include which have the invariant mass $M_{2,3}$ outside the ρ region, *i.e.* only events with $m_\rho + \Gamma/2 < M_{2,3}$ or $M_{2,3} < m_\rho - \Gamma/2$ are included. The two histograms show a much cleaner $7-2\cos(\phi)^2$ (x-polarization) and $7-2\sin(\phi)^2$ (y-polarization) behavior.